DESIGN THEORY FOR A FULLY SUPERCONDUCTING
SYNCHRONOUS MOTOR OR GENERATOR*

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DESIGN THEORY FOR A FULLY SUPERCONDUCTING SYNCHRONOUS MOTOR OR GENERATOR*

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ABSTRACT

Theory is developed for a superconducting synchronous motor
or generator having both the field and the armature windings super-
conducting. An approximate design is given for a 4-pole 30 megawatt
motor operating at 180 rpm.

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INTRODUCTION

The possibility of a high torque rotating machine in which both the field and the armature winding are superconducting has occasionally been suggested, and was recently discussed by the author.[1] A more detailed consideration is given here, along with a conceptual design. Although some of the major design problems for such a machine would undoubtedly be mechanical in nature, involving supporting structures, only the overall electric and magnetic design is presently considered. In the design of novel apparatus it is often instructive to examine the apparatus ab initio. For this reason the analysis is developed in terms of basic electromagnetic field theory, but with the following, more or less, standard simplifications: end effects in the machine are neglected, and the current in the windings is assumed to flow only along the machine axis, which is the z direction. Therefore, the magnetic field is two-dimensional in the cylindrical coordinates R and θ and assumed to arise from current sheets at the two windings, which implies that the thickness, or build-up, of each winding is small compared with its diameter.

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[1] Westinghouse Research Report 73-9J2-MACON-R1.

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For a three phase machine, A, B, C, with p pairs of poles, assume an armature winding density for the A phase (number of conductors per unit distance along the circumferential direction on the armature winding) given by

$$n_A = n_0 \left| \sin p\, \theta \right| \quad . \tag{1}$$

These conductors all run along the z axis and the region of negative $\sin p\, \theta$ is the return path for the current in the region of positive $\sin p\, \theta$. For a non-ideal winding different from (1), Fourier analysis must be used and $\sin p\, \theta$ is a principal Fourier component of the conductor density. For simplicity, only positive sequence fields will be considered, since negative sequence can be treated in the same manner, and the magnitude of the current flowing in each conductor in the A phase is assumed, as a function of time, to be

$$I_A = I_0 \cos \omega t \quad . \tag{2}$$

The conductor current along the positive z axis is

$$I_A \frac{\sin p\, \theta}{\left| \sin p\, \theta \right|} \quad . \tag{3}$$

For the phases B and C

$$n_B = n_0 \left| \sin \left(p\, \theta - \frac{2\pi}{3} \right) \right| \tag{4}$$

$$I_B = I_0 \cos \left(\omega t - \frac{2\pi}{3} \right)$$

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$$n_C = n_o \left| \sin \left(p\,\theta + \frac{2\pi}{3} \right) \right| \tag{5}$$

and

$$I_C = I_o \cos \left(\omega t + \frac{2\pi}{3} \right) \quad . \tag{6}$$

The surface current density of the armature winding, treated as a current sheet, is

$$J_s = n_A I_A \left| \frac{\sin p\,\theta}{\sin p\,\theta} \right| + n_B I_B \left| \frac{\sin \left(p\,\theta - \frac{2\pi}{3} \right)}{\sin \left(p\,\theta - \frac{2\pi}{3} \right)} \right|$$

$$+ n_C I_C \frac{\sin \left(p\,\theta + \frac{2\pi}{3} \right)}{\left| \sin \left(p\,\theta + \frac{2\pi}{3} \right) \right|} \tag{7}$$

where the subscript s is used because the armature is assumed to be the stator. Since from trigonometric identities it can be shown that

$$\frac{3}{2} \quad \sin (x - y) = \sin x \cos y + \sin \left(x - \frac{2\pi}{3} \right) \tag{8}$$

$$\cos \left(y - \frac{2\pi}{3} \right) + \sin \left(x + \frac{2\pi}{3} \right) \cos \left(y + \frac{2\pi}{3} \right)$$

substitution of (1) through (6) into (7) leads to

$$J_s = J_{so} \sin (p\,\theta - \omega t) \tag{9}$$

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where

$$J_{so} = \frac{3}{2} n_o I_o \quad . \tag{10}$$

The magnetic field of such a current distribution is well-known. If R_s is the radius of the stator winding, for $R < R_s$

$$\underset{\sim}{H}_s = \frac{2\pi J_{so}}{p R_s^{p-1}} \quad \nabla R^p \cos (p\theta - \omega t) \tag{11}$$

and for $R > R_s$

$$\underset{\sim}{H}_s = - \frac{2\pi J_{so}}{p} R_s^{p+1} \nabla \frac{1}{R^p} \cos (p\theta - \omega t) \quad . \tag{12}$$

The magnitude of the field vector is

$$H_s = 2\pi J_{so} \left(\frac{R}{R_s} \right)^{p-1} \tag{13}$$

for $R < R_s$

$$H_s = 2\pi J_{so} \left(\frac{R_s}{R} \right)^{p+1} \tag{14}$$

for $R > R_s$.

The total number of turns in each phase of the winding is

$$N = \frac{n_o}{2} \int_0^{2\pi} |\sin p\,\theta|\ R_s\ d\theta = 2\,n_o\,R_s \quad , \tag{15}$$

being one-half the integrated number of conductors.

Current and Field for the Field Winding

Assume a fixed symmetry axis in the cross-section of the rotor from which the angle $\bar{\theta}$ is measured, so that the cylindrical coordinates in the rotor reference frame are R and $\bar{\theta}$. In terms of the angle θ in the stator reference frame

$$\bar{\theta} = \theta - \phi \tag{16}$$

where

$$\phi = \phi_o + \int_0^t \omega_r\ dt \tag{17}$$

with ω_r the angular velocity of the rotor, and ϕ_o a constant. Assume a surface current distribution in the field winding current sheet on the rotor given by

$$J_r = J_{ro}\ \sin p\ \bar{\theta} \tag{18}$$

If R_r is the radius of the field winding, the magnetic field produced by this winding is, for $R < R_r$

$$H_r = \frac{2\pi\,J_{ro}}{p\,R_r^{p-1}} \; \nabla\,R^p\,\cos p\,\bar{\theta} \tag{19}$$

and for $R > R_r$

$$H_r = -\frac{2\pi\,J_{ro}}{p}\;R_n^{p+1}\;\nabla\,\frac{1}{R^p}\;\cos p\,\bar{\theta} \quad. \tag{20}$$

Power and Torque

The magnetic force exerted by the stator winding on the current in a unit element of length in the rotor is $J_r \times H_s$ in e.m.u., and the torque is $r \times (J_r \times H_s)$ and the power $\omega_r \cdot r \times (J_r \times H_s)$. Since ω_r is along the z axis, the expression for power reduces to $\omega_r\,R\,J_r(H_s)_R$ and the total power delivered through the rotor shaft per unit length of machine axis is

$$P = \omega_r\,R_r \int_0^{2\pi} J_r(H_s)_R\,R_r\,d\theta \tag{21}$$

which reduces to

$$P = 2\pi^2\,\omega_r R_r^2\,J_{so}J_{ro}\left(\frac{R_r}{R_s}\right)^{p-1}\sin(\omega t - p\phi) \quad. \tag{22}$$

Alternate expressions for the power obtained by substituting the magnetic fields for the current densities are also instructive. For example

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$$P = \frac{\omega_r}{2} R_s^2 H_s(R_s) H_r(R_s) \sin(\omega t - p \phi) \tag{23}$$

where $H_r(R_s)$ is the magnitude of the field of the rotor at the stator winding, and $H_s(R_s)$ is the magnitude of the stator field evaluated just outside or inside the stator radius, since the field $\underset{\sim}{H}_s$ has a discontinuity at R_s due to the current sheet.

Under steady state operation ω_r and P must be constant and it follows from (17) and (22) that

$$\omega_r = \frac{\omega}{p} \tag{24}$$

and $\sin(\omega t - p \phi) = -\sin p \phi_0,$ \hfill (25)

where, as defined here, ϕ_0 is negative for a motor and positive for a generator.

In general ω_r is determined by the equation of motion of the rotor. The total electromagnetic torque about the z axis is $\ell\, P/\omega_r$ where ℓ is the machine length, and if M is the moment of inertia connected to the rotor shaft and T is the load torque in the case of a motor or the applied torque in the case of a generator, the equation of motion of the rotor, assuming slow changes in time over a period of several cycles so that transients do not occur and P is still given by (22) with I_0 a functions of time, is

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$$M \dot{\omega}_r = 2\pi^2 \ell R_r^2 J_{so} J_{ro} \left(\frac{R_r}{R_s} \right)^{p-1} \sin(\omega t - p\phi) + T \qquad (26)$$

with the dot a time derivative.

Voltage

Since the voltage induced around a closed path is $-\int \dot{\underline{H}} \cdot d\underline{A}$, with $d\underline{A}$ an element of area, the voltage induced in a turn on the stator with one side at θ and the other at $\theta + \pi/p$, spanning one pole pitch, is

$$- \ell R_s \int_{\theta}^{\theta + \pi/p} \dot{H}_R \, d\theta. \qquad (27)$$

Between θ and $\theta + d\theta$ there are $n_A R_s d\theta$ conductors, which if multiplied by the voltage per turn and integrated between θ and π/p gives the voltage per pole pair. Consequently the terminal voltage for phase A is

$$V_A = -p \ell R_s^2 \int_{o}^{\pi/p} n_A \int_{\theta}^{\theta + \pi/p} \dot{H}_R(\theta') \, d\theta' \, d\theta \qquad (28)$$

where the contribution to H_R from the rotor gives the "internal" voltage and the contribution from the stator field gives the reactive drop. After the integrations in (28) are performed the voltage can be put in the following form with the aid of (10) and (15):

$$V_A = -\frac{\pi}{2} \omega_r \ell R_s N H_r(R_s) \sin p\phi + \frac{3 \pi^2 \ell N^2}{4 p} \dot{I}_A \qquad (29)$$

assuming J_{ro} to be constant. In steady state the ratio of the magnitude of these two terms can be shown to be just $H_r(R_s)/H_s(R_s)$. From the last term on the righthand side it follows that the synchronous reactance is

$$X_s = \frac{3 \pi^2 \ell N^2 \omega}{4 p} \qquad , \qquad (30)$$

Magnetic Shield

If a ferromagnetic shield is placed around the machine, the fields will be altered by an amount which is difficult to calculate, since, in general, parts of the shield will be saturated. While the field due to the shield is, by no means, negligible, it will be less than the field of the currents, and may be neglected as the first step in an interation process.

The maximum flux per unit axial length which can be shielded is $(R_0 - R_i) B_s$, where R_0 is the outside radius and R_i the inside radius of the shield, and B_s is the saturation flux density. Since the flux is along the θ direction, it follows that $(R_0 - R_i) B_s$ must be greater than the integrated θ component of flux density, from zero to R_i, inside the shield. Due to the rotor the latter is

$$\int_0^{R_1} (H_r)_\theta \, dR \tag{31}$$

which has the maximum value

$$\frac{2\pi \, J_{ro} \, R_r}{p} \left(\frac{R_r}{R_1} \right)^p = \frac{R_i}{p} \, H_r(R_1) \quad . \tag{32}$$

Due to the stator a similar contribution is obtained, and in the worst possible case where the stator and rotor fields are additive

$$(R_0 - R_1) \geq \frac{R_i}{B_s p} \left[H_r(R_1) + H_s(R_1) \right] \tag{33}$$

which gives an estimate of the thickness of the shield.

Approximate Machine Dimensions

If (33) is taken as an equality and R_i approximated by R_s, it gives for the outside machine radius

$$R_0 \simeq R_s \left[1 + \frac{H_r(R_s)}{B_s p} \, (1 + K) \right] \tag{34}$$

where K denotes the ratio, $H_s(R_s)/H_r(R_s)$. Since this ratio is the ratio of the voltage drop in the synchronous reactance to the internal voltage, for most applications K should be kept smaller than unity.

In terms of the total machine power ℓP, in watts, (23) gives for R_s in centimeters

$$R_s = \frac{1}{H_r(R_s)}\left(\frac{2 \ \ell P \times 10^7}{\ell \ K \ \omega_r \ \sin p \ \phi_o}\right)^{1/2} \quad . \tag{35}$$

The radius of the rotor R_r is obtained from R_s minus the gap width and the thickness of the windings. The number of turns may be obtained from (29).

As a particular design consider a four pole motor developing 30 megawatts with $\frac{\omega}{2\pi} = 6$ Hertz ($\frac{\omega_r}{2\pi} = 3$), and with $H_r(R_s) = B_s = 20000$ Oe, $K = 1/3$, $|\sin p \ \phi_o| = 0.7$ and $\ell = 160$ cm. Then $R_s = 45$ cm and $R_o = 75$. A reasonable value for R_r might be about 35 cm, leaving several centimeters for the winding thickness. The actual winding thickness will be determined by thermal and mechanical considerations. The relatively small thickness required by the conductors is given by dividing J_{ro} and J_{so} by the current densities assumed.

Conductor Design

Previous calculations[1] for the ac loss in the stator, for a design similar to that considered here, give for the ratio of refrigerator power, needed to remove the loss, to the machine output the approximate value

$$250 \ \frac{pd}{R_s}\left(\frac{\alpha j_c}{j_{so}}\right) \tag{36}$$

where α j_c/j_{so} is the ratio of the critical current density for the stator winding to the actual density, and d is the diameter of the filaments in a filamentary superconducting wire. In commercial wire d is frequently five microns. It seems reasonable to expect that a factor of two improvement, i.e., a d of 2.5×10^{-4} cm, is achievable, and with this value along with the assumption that the operating current density is one-half its critical value, the ratio (36) becomes 0.13 p/R_s, which for p = 2 and $R_s \approx 45$ gives 0.006. For the 30 megawatt machine this represents a refrigerator input power of about 180 kilo-watts.

To obtain this level of ac loss in the armature conductor a diameter D of about 3 mils is required for the multifilamentary wire. Such a wire at one-half its critical current density might be expected to carry in the neighborhood of 5 amperes. Since the the peak amperes of the conductor, if a voltage per phase of 10,000 R.M.S. is assumed,

would be 2000 amperes (for roughly 0.7 power factor), then about 400 wires would be needed per conductor. These wires may, for example, be bundled together in the manner discussed by the author and M. S. Walker.[2] The loss within the filamentary wire would be about 80 milliwatts/cc.

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[2]Westinghouse Patent Disclosure.